Accepted paper with abstract, SoCG'09

Naoki Katoh and Shin-ichi Tanigawa

A Proof of the Molecular Conjecture

A d-dimensional body-and-hinge framework is, roughly speaking, a structure consisting of rigid bodies connected by hinges in d-dimensional space, whose generic infinitesimal rigidity has been characterized in terms of the underlying multigraph independently by Tay and Whiteley as follows: A multigraph G can be realized as an infinitesimally rigid body-and-hinge framework by mapping each vertex to a body and each edge to a hinge if and only if $(D - 1)G$ contains D edge-disjoint spanning trees, where $D = (d + 1 \text{ choose } 2)$ and $(D - 1)G$ is the graph obtained from G by replacing each edge by $(D - 1)$ parallel edges. In 1984 they jointly posed a question about whether their combinatorial characterization can be further applied to a nongeneric case. Specifically, they conjectured that G can be realized as an infinitesimally rigid body-and-hinge framework if and only if G can be realized as that with the additional ``hinge-coplanar'' property, i.e., all the hinges incident to each body are contained in a common hyperplane. This conjecture is called the Molecular Conjecture due to the equivalence between the infinitesimal rigidity of ``hinge-coplanar'' body-and-hinge frameworks and that of bar-and-joint frameworks derived from molecules in 3-dimension. In 2-dimensional case this conjecture has been proved by Jackson and Jordán in 2006. In this paper we prove this long standing conjecture affirmatively for general dimension. Also, as a corollary, we obtain a combinatorial characterization of the 3-dimensional rigidity matroid for the bar-and-joint framework of the square of a graph.